Math 2550 - Homework # 8

Eigenvalues and Eigenvectors

1. For each matrix A do the following: (i) Find the eigenvalues of A, (ii) Find a basis for each eigenspace $E_\lambda(A)$, (iii) For each eigenvalue, compute it's algebraic and geometric multiplicity.

   (a) $A = \begin{pmatrix} 3 & 0 \\ 8 & -1 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

   (d) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

   (e) $A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$

   (f) $A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

2. Let A be an $n \times n$ matrix. Suppose that λ is an eigenvalue of A with corresponding eigenvector $\vec{x}$. Find a formula for $A^n\vec{x}$ for any $n = 1, 2, 3, 4, \ldots$.

3. Let A be an $n \times n$ matrix and λ be an eigenvalue of A. Prove that $E_\lambda(A)$ is a subspace of $\mathbb{R}^n$.